Student Study Session Topic: Rate and Accumulation Questions

The integral of a rate of change gives the (net) amount of change. The general form of the equation is $F(x) = F(x_0) + \int_{x_0}^{x} F'(t)\,dt$,where $x = x_0$ is the initial time, and $F(x_0)$ is the initial value. Since this is one of the main interpretations of the definite integral the concept may come up in a variety of situations. In a sense all integrals are accumulators. Particle motion problems can often be approached as accumulation problems.

What should you know how to do?

- Understand the question. It is often not necessary to do as much computation as it seems at first.
- Often these problems contain a lot of writing. Be ready to read, think and apply.
- Understand that "rate" indicates a derivative. The units of measure also indicate derivative (miles per hour, gallons per minute, etc.) since you're working with a derivative, expect to integrate to find the amount.
- Typically the first part of the question asks for an amount, integrate (on your calculator) for the first 2 or three points. 2000 AB4 (a), 2005 AB2 (a), 2006 AB 2(a)
- You may then be asked to write or work with a function that will look like $F(x) = F(x_0) + \int_{x_0}^{x} f(t)\,dt$. If appropriate the derivative is found easily using the Fundamental Theorem of Calculus: $F'(x) = f(x)$.
- You may be asked to explain the meaning of a derivative or definite integral or its value in terms of the context of the problem.
- In-out problems: 2 rates of change work together 2000 AB4, 2005 AB 2
- Max/min and increasing/decreasing analysis 2000 AB 4, 2005 AB 2
- Multiple-choice questions may give the information needed in a graph or in a table.

Student Notes

You've been doing this since Algebra 1: Slopes are rates of change. If *f* is a constant function and if $f(x)=m$, then $F(x)=F(x_0)+\int_{x_0}^{x} m\,dt = F(x_0)+m(x-x_0)$ - a line. And since, for a line, $F'(x)=m$ this can be written as $F(x)=F(x_0)+F'(x_0)(x-x_0)$. Look familiar?

Multiple-choice answers: 1 B, 2 D, 3 E, 4 A, 5 A, 6 A, 7 D, 8 E.

WATCH and LISTEN to the multiple-choice questions being solved

Go to http://tinyurl.com/NMSI-Math-10 Click on the "Full Screen" arrow. Then click anywhere on the page to see and hear from that point on.

Click anywhere to go back anytime.

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

   (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

   (b) How many gallons of water are in the tank at time $t = 3$ minutes?

   (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

   (d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

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(a) Method 1: $\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2} \Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$\frac{dL}{dt} = \sqrt{t+1}$; $L(t) = \frac{2}{3}(t+1)^{3/2} + C$

$L(0) = 0$: $C = -\frac{2}{3}$

$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}$; $L(3) = \frac{14}{3}$

Method 1:
$3 \begin{cases} 2 : \text{definite integral} \\ \quad 1 : \text{limits} \\ \quad 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

– or –

Method 2:
$3 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{solves for } C \text{ using } L(0) = 0 \\ 1 : \text{answer} \end{cases}$

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

1 : answer

(c) Method 1:

$$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right) dx$$
$$= 30 + 8t - \int_0^t \sqrt{x+1}\,dx$$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$
$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$
$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$
$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

Method 1:
$2 \begin{cases} 1 : 30 + 8t \\ 1 : -\int_0^t \sqrt{x+1}\,dx \end{cases}$

– or –

Method 2:
$2 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{answer} \end{cases}$

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$

$A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

$3 \begin{cases} 1 : \text{sets } A'(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{justification} \end{cases}$

2. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

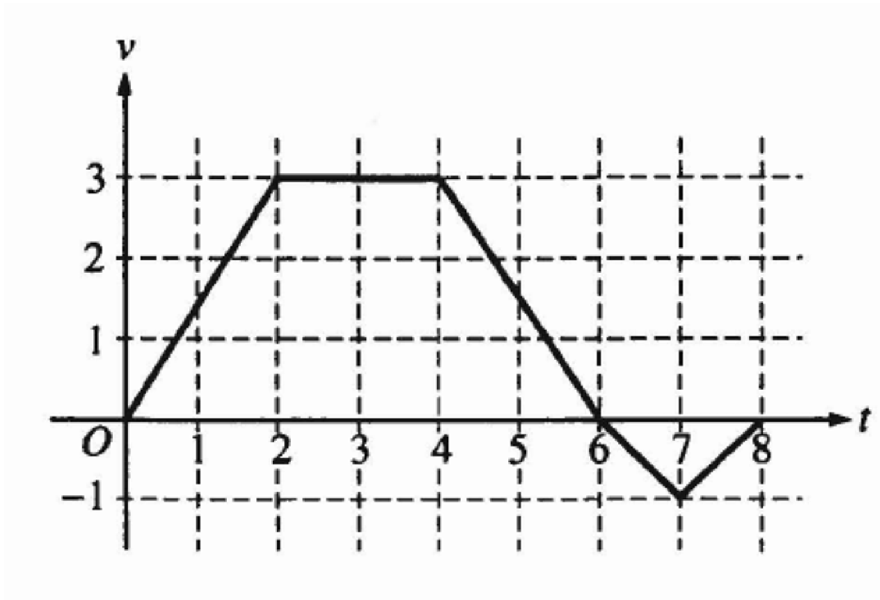
(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

WRITE ALL WORK IN THE TEST BOOKLET.

Name: ________________________ Class: ___________________ Date: __________ ID: A

Rate and Accumulation Prep Session Questions

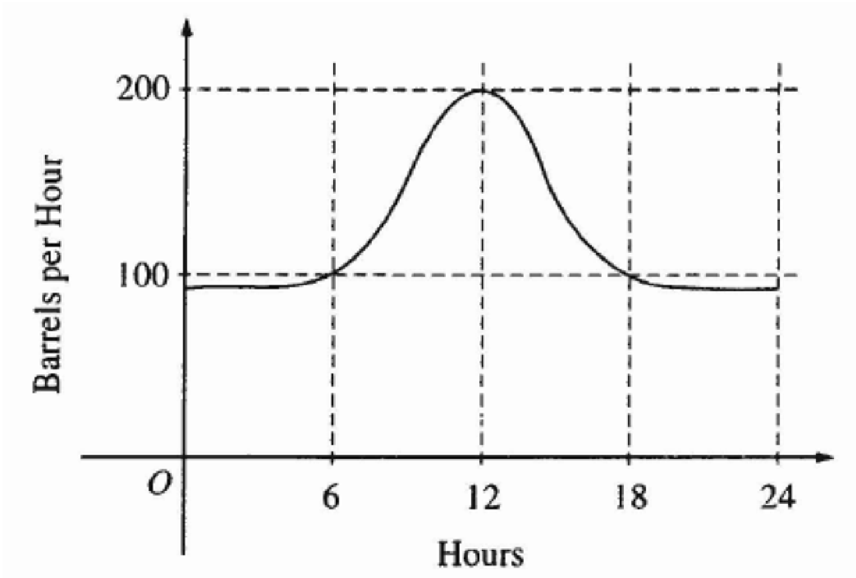


A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

____ 1. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

a. 14
b. 13
c. 11
d. 8
e. 6

____ 2.



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

a. 500
b. 600
c. 2,400
d. 3,000
e. 4,800

____ 3.

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at t =0 is 11 feet per second, the approximate value of the velocity at t = 6, computed using a left-hand Riemann sum with three subintervals of equal length, is

a. 26 ft/sec
b. 30 ft/sec
c. 37 ft/sec
d. 39 ft/sec
e. 41 ft/sec

____ 4. Insects destroyed a crop at the rate of $\dfrac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

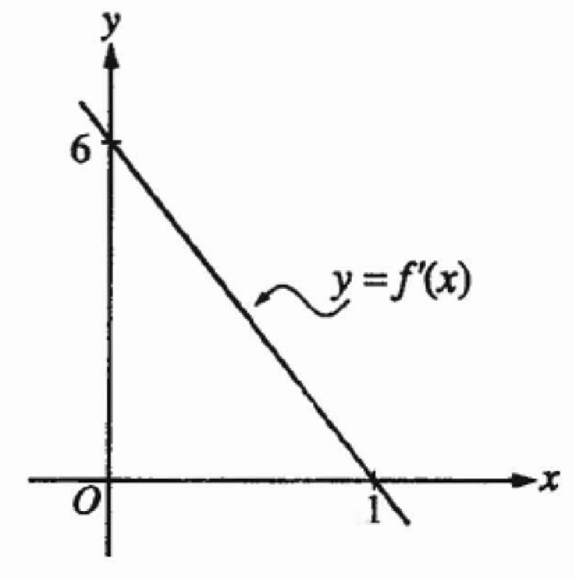
a. 125
b. 100
c. 88
d. 50
e. 12

____ 5. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

a. $\int_{1.572}^{3.514} r(t)dt$
b. $\int_{0}^{8} r(t)dt$
c. $\int_{0}^{2.667} r(t)dt$
d. $\int_{1.572}^{3.514} r'(t)dt$
e. $\int_{0}^{2.667} r'(t)dt$

____ 6. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time t =0 minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time t = 5 minutes?

a. 112°F
b. 119°F
c. 147°F
d. 238°F
e. 335°F

____ 7. 

The graph of *f* ′, the derivative of *f*, is the line shown in the figure above. If *f*(0) = 5. then *f*(1) =

a. 0
b. 3
c. 6
d. 8
e. 11

____ 8. A particle moves along the x-axis so that at any time $t > 0$. its acceleration is given by $a(t) = \ln\left(1 + 2^t\right)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

a. 0.462
b. 1.690
c. 2.555
d. 2.886
e. 3.346

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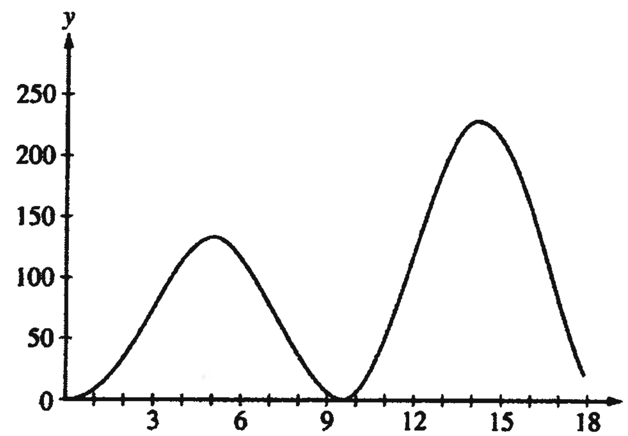
Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.



(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars

$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$
Let $R = 12.42831$ and $S = 16.12166$
$L(t) \ge 150$ for t in the interval $[R, S]$

$\frac{1}{S-R}\int_R^S L(t)\,dt = 199.426$ cars per hour

$3 : \begin{cases} 1 : t\text{-interval when } L(t) \ge 150 \\ 1 : \text{average value integral} \\ 1 : \text{answer with units} \end{cases}$

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$$\int_{13}^{15} L(t)\,dt = 431.931 > 400$$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \ge 200$ on that interval.
$L(t) \ge 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

Yes, a traffic signal is required.

$4 : \begin{cases} 1 : \text{considers 400 cars} \\ 1 : \text{valid interval } [h, h+2] \\ 1 : \text{value of } \int_h^{h+2} L(t)\,dt \\ 1 : \text{answer and explanation} \end{cases}$

OR

$4 : \begin{cases} 1 : \text{considers 200 cars per hour} \\ 1 : \text{solves } L(t) \ge 200 \\ 1 : \text{discusses 2 hour interval} \\ 1 : \text{answer and explanation} \end{cases}$